Physics IV
ISI B.Math
Midterm Exam : March 8, 2022

Total Marks: 60
Time : 2.5 hours
Answer all questions

1.(**Marks: 2 + 1 + 1 + 1 + 1 + 2 + 2 =10**)

In an inertial frame of reference four events A, B, C and D have the following coordinates

$A : x_A = 0, t_A = 0$
$B : x_B = L, t_B = \frac{2L}{c}$
$C : x_C = L, t_C = -\frac{2L}{c}$
$D : x_D = 0, t_D = \frac{2L}{c}$

Here $L > 0$ and you can ignore the y and z directions.(b),(c) , (d) and (e) must be acoompanied by a one or two line brief explanation

(a) Draw a spacetime diagram showing events A, B and C. Draw and label the regions that constitute the past, future and elsewhere relative to A

(b) Could A have caused event B?

(c) Could A have caused event C ?

(d) Could C have caused event A ?

(e) Could C have caused event B?

(f) Now draw a spacetime diagram showing events A and D only. Given that your worldline passed through A and D and given that your speed during the intervening time never exceeds c, draw and shade the region that you might have visited during the time between A and D.

(g) Draw a new diagram again showing A and D only. This time, draw and shade the region of spacetime consisting of events which could not have affected you at A but could have affected you at D.

2. (**Marks : 3 + 3 + 3 + 3 = 12**)

(a) Show that the the sum of two future-pointing null four-vectors is either timelike or null, and is again future-pointing. Under what conditions is the sum null?

(b) Every four-vector orthogonal to a timelike vector is spacelike. (Two four vectors A^μ and B^μ are orthogonal if $A^\mu B_\mu = B^\mu A_\mu = 0$ $\mu = 0, 1, 2, 3$)

(c) If P^μ is the four-momentum of a particle and F^μ is the four-force defined as the product of the mass and the four-acceleration , show that $P^\mu F_\mu = 0$.

(d) Using the result in (c) , show that the four force F can be written as $F = (\frac{\gamma}{c}\mathbf{f}\cdot\mathbf{v}, \gamma\mathbf{f})$ where $\mathbf{v}$ is the three-velocity of the particle and $\mathbf{f}$, the three force $= \frac{d\mathbf{p}}{dt}(\mathbf{p} = m\gamma\mathbf{v}$. $\mathbf{p}$ being the relativistic three momentum).

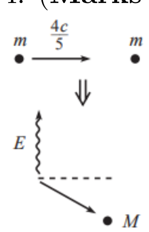
3. (**Marks : 3 + 3 + 6 = 12**)

(a) In special relativity it is possible for the order of two events to be reversed in another inertial frame. Does that imply that there exists an inertial frame in which you can get off a bus before you can get on it ? Explain briefly.

(b) A train moves at speed $\frac{4c}{5}$. A clock is thrown from the back of the train to the front. As measured in the ground frame, the time of flight is 1 sec. Is the following reasoning correct ? " The γ factor between the train and the ground frame is $\frac{5}{3}$. Since moving clocks run slow, the time elapsed on the clock during the flight is $\frac{3}{5}$ second."

(c) A and B both start at the origin and simultaneously head off in opposite directions at speed v with respect to the ground. A moves to the right and B moves to the left. Consider a mark on the ground at $x = L$. As viewed in the ground frame, A and B are at a distance $2L$ apart when A passes this mark. As viewed by A, how far away is B when A coincides with this mark ?

4. (**Marks : 7 + 4 + 5 = 16**)



(a) A mass m moving at speed $\frac{4}{5}c$ collides with another mass m at rest. The collision produces a photon with energy E traveling perpendicular to the original direction, and a mass M traveling in another direction as shown in the figure. In terms of E and m,what is M? What is the largest value of E (in terms of m) for which this setup is possible ?

(b) Write down a relativistic expression for the kinetic energy of a particle of mass m and speed v. Show that it reduces to the usual Newtonian expression in the appropriate limit.

(c) It is given that the x component of the four momentum of a particle is zero in all inertial frames. The claim is that the momentum four vector p^μ must have the form $(0, 0, 0, 0)$. Justify this claim.

5. (**Marks : 4 + 3 + 3 = 10**)

(a) An observer in inertial frame S measures a charge density ρ and current density $\mathbf{j}$ in his frame. An observer in frame S' moving with a velocity v with respect to S along the common $x - x'$ axis measures a charge density ρ' and current density $\mathbf{j}'$ in his frame for the same charge and current distribution. How are the quantities $\rho', \mathbf{j}'$ related to the corresponding quantities in the S frame ?

Which combination of ρ and $\mathbf{j}$ remains invariant under a Lorentz transformation ?

The electric and magnetic fields $(\mathbf{E}, \mathbf{B})$ are measured with respect to an observer in an inertial frame S . It can be shown that i) $\mathbf{E} \cdot \mathbf{B}$ and ii) $E^2 - c^2 B^2$ are invariant quantities under Lorentz transformations.

(b) Show that a pure electric field in one inertial frame cannot be transformed into a pure magnetic field in another inertial frame.

(c) A particular electromagnetic field has its $\mathbf{E}$ field at an angle θ_0 to its $\mathbf{B}$ field, and θ_0 is invariant to all observers. What is the value of θ_0 ?